Modeling epidemic spreading in star-like networks

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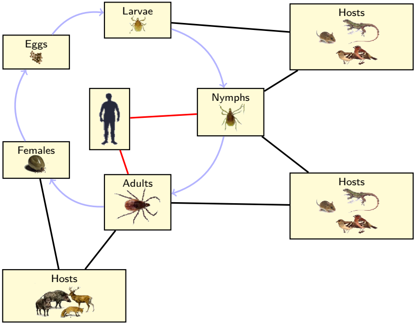
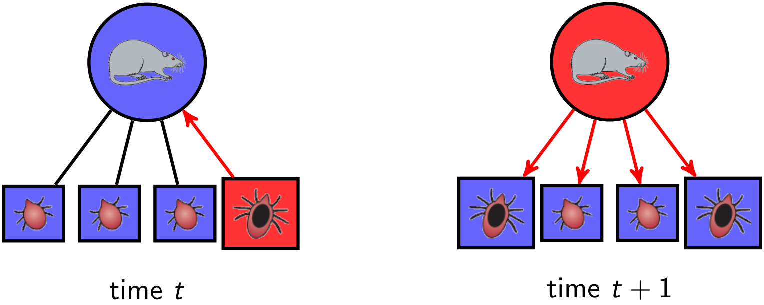
Tick-Borne Encephalitis

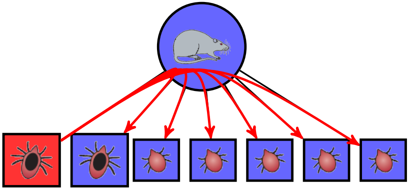
- endemic in Eurasia from Europe, through Russia To China and Japan
- the virus causes potentially fatal neurological infecion
- in last years emergenge of the virus in new area and increase of morbidity
- maintained in nature by complex cycle involving Ixodid ticks (*I. ricinus* and *I. persulcatus*) and wild vertebrate hosts



Systemic Transmission



Non-Systemic Transmission



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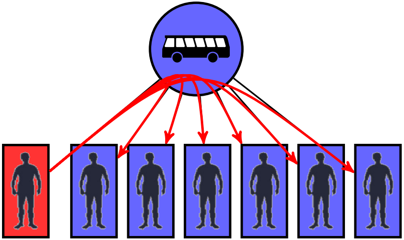

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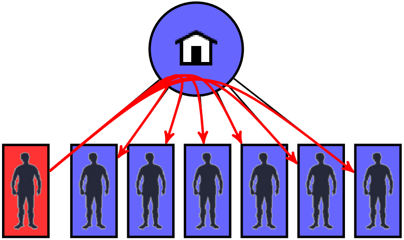


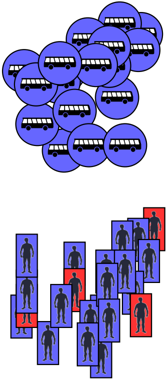
Our research question

how do the non-systemic transmission together with the different aggregation patterns influence the pathogen spreading?

Spreading Model

- at time t a fraction, $\pi(t)$, of passengers (ticks) are infectious

time t



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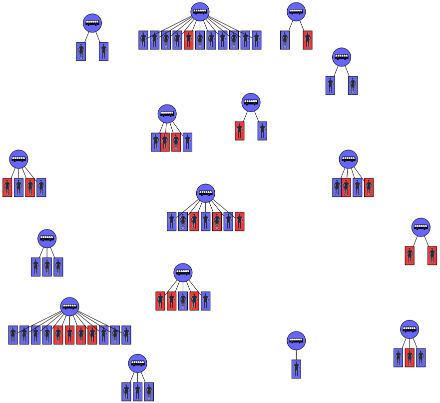

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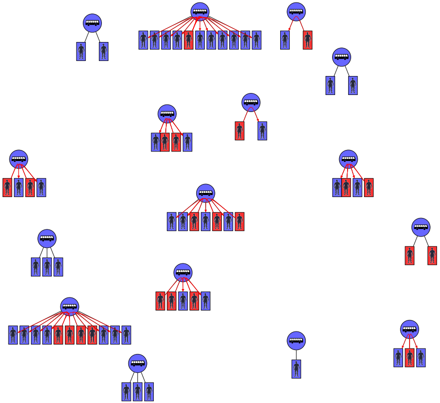

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- β transmission probability for infectious path
- μ recovery probability

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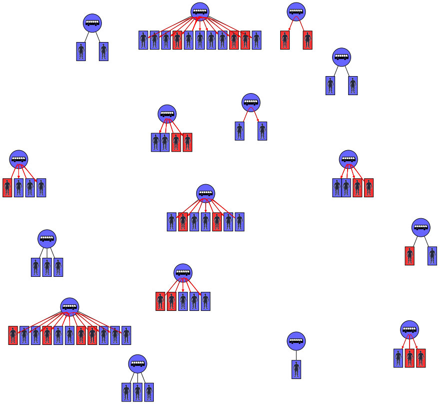

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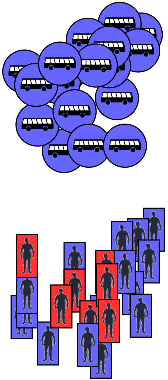
$$\Rightarrow \pi(t+1) = f(\pi(t))$$

time $t+1$



Analytical Framework

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Let $\pi(t)$ be the prevalence of infection among passengers at time t, the probabilityy for a susceptible passenger on a bus transporting k individuals including himself to be infectious at time $t + 1$ is

$$1 - (1 - \beta)^{(k-1)\cdot\pi(t)}$$

Math

Recalling that $\mathbb{P}(k)$ is the probability for a bus to have k passengers, the probability for a passenger to be on a k-bus is

$$\frac{\#\text{passengers on a } k\text{-bus}}{\#\text{passengers}} = \frac{k \cdot \#k\text{-bus}}{\#\text{passengers}} =$$
$$= k \cdot \frac{\#k\text{-bus}}{\#\text{bus}} \cdot \frac{\#\text{bus}}{\#\text{passengers}} = k \cdot \mathbb{P}(k) \cdot \frac{1}{\langle k \rangle}$$

Math

thus, the probability for a susceptible passenger at time t to be infectious at time $t+1$ is

$$\sum_{k=1}^{\infty}\left[1-(1-\beta)^{(k-1)\cdot\pi(t)}\right]\cdot\frac{k}{\langle k\rangle}\cdot\mathbb{P}(k)$$

and therefore the prevalence among passenger at time $t+1$ is

$$\pi(t+1)=f(\pi(t))=$$

$$=(1-\mu)\cdot\pi(t)+[1-\pi(t)]\cdot\left\{1-\sum_{k=1}^{\infty}(1-\beta)^{(k-1)\cdot\pi(t)}\cdot\frac{k}{\langle k\rangle}\cdot\mathbb{P}(k)\right\}$$

Equilibria

imposing the stationary condition $\pi(t+1) = \pi(t) = x$ we can derive the equilibria as solutions of the following equation

$$x = f(x) = (1-\mu)\cdot x + [1-x]\cdot\left\{1 - \sum_{k=1}^{\infty}(1-\beta)^{(k-1)\cdot x}\cdot\frac{k}{\langle k\rangle}\cdot\mathbb{P}(k)\right\}.$$

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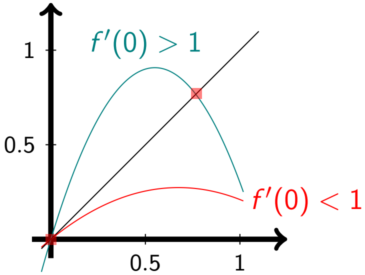

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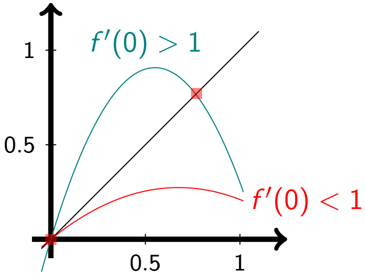

therefore conditions to have one, and only one, solution $\hat{x} \in (0,1)$ is that $f'(0) > 1$ or

$$-\frac{\ln(1-\beta)}{\mu} > \frac{\langle k\rangle}{\langle k^2\rangle - \langle k\rangle}.$$

Stability

recalling that $f''(x) < 0$ and that

- $f'(1) = -\mu + \sum_k (1-\beta)^{k-1} \frac{k}{\langle k \rangle} \mathbb{P}(k) > -\mu > -1$
- $f'(\hat{x}) < 1$

hence $\hat{x}$ is asymptotically stable when it exists. Therefore:

- disease-free equilibrium is asymptotically stable when $\hat{x}$ does not exist.
- disease-free is unstable when $\hat{x}$ exists. Furthermore, when $\hat{x}$ exists it is also asymptotically stable.

Conclusion and Discussions

- co-feeding transmission
- spreading on star-like networks
- spreading dynamic on dynamic bipartite networks
- analytical result confirmed by simulations

Acknowledgements

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